Mathematics Guidance for Calculations

2021-2022

Rationale

This guidance offers schools a foundation for the teaching of calculations across EYFS and Primary. It is designed to support practitioners with the core strategies and progression needed for all children to be successful.

It is hoped that this guidance will complement and strengthen current provision for mathematics. Being able to calculate is an essential and significant part of being an effective mathematician.

In January 2020, arithmetic assessments and an audit of calculation strategies showed that the use of standard formal algorithms were limiting conceptual understanding in children. Evidence showed that children would automatically use a formal algorithm even if it was not the most efficient way of solving a particular calculation.

% of children in the cluster correctly answering an age related question for addition	% of children in the cluster correctly answering an age related question for subtraction	% of children in the cluster correctly answering an age related question for multiplication	% of children in the cluster correctly answering an age related question for division
65%	54%	52%	43%

Table 1: Calculation Audit data: January 2020

This evidence resulted in the development of this new calculation guidance in February 2020. The strategies included in this guidance have been selected because they represent the most accurate and efficient methods for children to understand all areas of mathematics. Often the more ‘formal’ methods close down thinking, do not support the teaching and learning of place value and only have a small level of accountability in the National Curriculum tests.

Every school carries out a calculation audit in January and June. Data is collected and analysed at both school and Trust level. This data will help to inform and review this calculations guidance and the wider curriculum on a regular basis.

Aims

This calculation guidance aims to ensure all children:

- understand important concepts and make connections within mathematics;
- show high levels of fluency in performing written and mental calculations;
- are taught consistent calculation strategies;
- are ready for the next stage of learning;
- have a smooth transition between phases;
- are able to add, subtract, multiply and divide efficiently;
- are competent in fluency, reasoning and problem solving.

Aims of the National Curriculum

The aims of the National curriculum 2014 and aim to ensure that all children:

- become **fluent** in the fundamentals of mathematics, through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately

- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language

- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Pupils should make connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems (NC 2014).

Fluency

"Fluency is about being flexible"

Pupils should be able to:

- count accurately
- use a variety of counting strategies (e.g. one to one, cardinality, order irrelevance);
- choose a variety of manipulatives/equipment to support their understanding;
- calculate effectively;
- recognise and use arithmetic laws to help with their calculations (commutative, associative, distributive, inverse, BODMAS);
- demonstrate they understand the importance of equivalence;
- recognise the structure of number (e.g. odd, even, prime, multiples, factors, square, triangular, Fibonacci, negative, natural, fraction, decimal, percentage);
- recognise mathematical symbols;
- use place value effectively (including zero);
- choose when to calculate mentally or use a written strategy;
- notice numbers within numbers (number bonds, partitioning);
- subitise; (recognise the number in a group without needing to count them);
- use rounding and adjusting;
- use doubling and halving.

Examples of Fluency include;

Fact Families
Starting with the knowledge that 2 + 4 = 6, pupils will know the 7 other related facts:

2 + 4 = 6	4 + 2 = 6	6 – 4 = 2	6 – 2 = 4
6 = 2 + 4	6 = 4 + 2	2 = 6 - 4	4 = 6 - 2

Associated Facts
Starting with the knowledge that 8 x 5 = 40, pupils who demonstrate fluency, reasoning and problem solving skills are able to use this fact to create others such as:

5 x 8 = 40	50 x 8 = 400	16 x 2.5 = 40	0.8 x 0.5 = 0.4
8 x 5 = 40	80 x 50 = 4000	40 x 8 ≠ 5	5 x 8 = 10 x 4
40 ÷ 5 = 8	8 x 5 = 20 x 2	5 x 8 = 8 + 8 + 8 + 8 +85 x 8 =	2^3x √25 = 40 = 8 x 5
40 ÷ 8 = 5	(2 x 4) x 5 = 10 x 4	(5 x 10) – (5 x 2)	40 = 8 x 5

In calculating 16 x 2.5, pupils would be able to use their doubling and halving skills (fluency) to justify (reasoning) that this is equivalent to 8 x 5 by pattern spotting (problem solving).

Variation and Structure

Carefully selected and sequenced questions expose pupils to pattern and structure. The examples in the columns below are similar in some ways and different in others. By thinking about what changes and what stays the same can help teachers to construct sets of examples that make it more likely that pupils will learn.

2x4=8	4x6=24	3x5=15
2x40=80	4 x 60=240	3x50=150
2x400=800	4x600= 2400	3x500=1500
20x4=80	60x40=2400	30x5=150
200x4=800	600 x 4 = 2400	300x5=1500
0.2x4 = 0.8	6x0.4=2.4	30x5=3x50
0.2 x0.4 =0.08	24÷4=6	0.3x0.5= 0.15
2x40= 20x4	240÷60=40	0.015 = 0.03 x 0.5

Key pedagogical questions to support understanding of variation and structure include;

- What is the same?
- What is different?
- What changes?
- What stays the same?
- If I change this thing, how do the other things change?

Reasoning

"Reasoning is about children thinking and thinking harder"

Pupils should be able to:

- offer conjectures (to offer and test a particular example to see what happens);
- describe and explain their thinking;
- convince someone else of their answer;
- justify with some logical arguments;
- prove whether they are right or wrong;
- specialise in order to test a particular example to see what happens;
- offer generalisations, even if they are incorrect;
- offer reasons for their thinking;
- notice connections;
- use a high level of mathematical language;
- use both additive and multiplicative reasoning (e.g. to make 2 into 10 you could add 8 – additive reasoning; or you could multiply 2 by 5 – multiplicative reasoning);
- decide if situations are always, sometimes or never true;
- interpolate and extrapolate information (e.g. Hannah is half way between 3m and 4m so I can interpolate her height is 3.5m, and I can extrapolate that she will be 5m in 2 years time);
- offer counter examples;
- ask 'what happens if …?';
- notice and characterise the problem.

NRICH 5 Stages of Reasoning (https://nrich.maths.org/11336)

Step one	Describing	Simply tells what they did.
Step two	Explaining	Offers some reasons for what they did. These may or may not be correct. The argument may yet not hang together coherently. This is the beginning of inductive reasoning.
Step three	Convincing	Confident that their chain of reasoning is right and may use words such as, 'I reckon' or 'without doubt'. The underlying mathematical argument may or may not be accurate yet is likely to have more coherence and completeness than the explaining stage. This is called inductive reasoning.
Step four	Justifying	A correct logical argument that has a complete chain of reasoning to it and uses words such as 'because', 'therefore', 'and so', 'that leads to' ...
Step five	Proving	A watertight argument that is mathematically sound, often based on generalisations and underlying structure. This is also called deductive reasoning.

How can we support children to develop reasoning?

- Modelling
- Mathematical language
- Sentence stems
- Group work
- Understanding how others work
- Personal notes & recording

Possible questions to ask …

- How do you know?
- How did you start?
- How could you prove it?
- What do you notice?
- Can you organise them logically to prove all the possibilities?

Problem Solving

"Problem solving is about having a strategy"

Pupils should be able to:

- show they can work systematically;
- check their answer using a different strategy;
- offer more than one solution;
- notice, create and extend pattern;
- record their thinking in a variety of ways, including using equipment;
- demonstrate resilience and perseverance in keeping going;
- work backwards;
- find all the possibilities;
- solve a variety of problems in different contexts (e.g. real life, fictitious, diagrams, words);
- use the bar model in order to understand a problem;
- use the language of part and whole, and known and unknown;
- use information given to work out information not given;
- classify different types of problems;
- critique and improve their own work and that of their peers.

NRICH suggests there are 4 stages to the problem-solving process (https://nrich.maths.org/10865)

The stages of the problem-solving process
The problem-solving process can usually be thought of as having four stages:

- Stage 1: Getting started
- Stage 2: Working on the problem
- Stage 3: Digging deeper
- Stage 4: Reflecting

Stage 1: Getting started will mean offering them strategies to help them engage with the problem. These could be prompts such as:

- Tell me/a partner what you think the problem is about.
- What would help you understand the problem?
- You might like to draw a diagram, act it out or represent it with a model.
- Could you try something out and see what happens?
- What other problems have you seen that are 'a bit like' this one?
- What mathematical skills have you got that could be helpful here?
- Try making a simpler case to get an idea of how the problem works.

Stage 2: Working on the problem will usually involve using one or several problem-solving skills such as:

- Trial and improvement
- Working systematically (and remember there will be more that one way of doing this: not just the one that is obvious to you!)
- Pattern spotting
- Working backwards
- Reasoning logically
- Visualising
- Conjecturing

Stage 3: Digging deeper usually happens when the problem has been explored and then it is possible to look for generalisations and proof.

Stage 4: Reflecting is the part of the problem-solving process where we support the children to:

- interpret their findings so far in the context of the problem
- explain their solution both verbally and in writing
- evaluate their method and compare different strategies.

Structure of the guidance

This calculations guidance is designed to be used from Early Years to at least the end of Key Stage 2. There are no levels, stages or ages attributed to each section. Rather it includes progression that all children need to access, whatever their starting points.

The strategies in this policy are informed by a range of research and a consideration of different strategies and approaches. Children are introduced to the processes of calculations through practical, oral and mental activities. As children begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods. They learn to interpret and use the signs and symbols involved. Children learn how to use manipulatives such as counters, to support their mental and informal written methods of calculation.

We recognise that there are many successful written methods in use today. However, we know from our evidence collected and research, that the emphasis should remain on conceptual understanding (relational learning) rather than procedural (instrumental) learning (Skemp 1989).

However, we would always encourage children to estimate their answer before calculating, whether they are using a mental or written strategy.

There are many misconceptions about which calculation strategies children should use. These misconceptions are around expectations from Ofsted, STA, DfE and other external bodies. The strategies included in this guidance have been selected because they represent the most accurate and efficient methods for children to understand all areas of mathematics. Often the more ‘formal’ methods close down thinking, do not support the teaching and learning of place value and only have a small level of accountability in the National Curriculum tests. In addition, the Ofsted framework does not seek any particular methods, but looks to ensure that children are offered age related mathematical tasks.

Representation and Structure

Part of being mathematically proficient involves conceptual understanding. This is the comprehension of mathematical concepts, operations and relations. Research has shown that pupils who use a conceptual approach develop a deeper understanding of mathematics than those who only rely on a procedural approach (knowing how but not why). A pupil with procedural understanding would be oblivious to the fact that an algorithm had been misapplied or incorrectly remembered and their understanding would be non-transferable (Bills, 1998). While pupils need a mathematical diet of both conceptual and procedural understanding it is our view for pupils in this Trust that conceptual understanding is the foundation upon which schemata is built.

At the heart of conceptual understanding is the CPA approach. This involves modelling mathematical concepts/ideas with concrete materials, manipulating these and developing the language associated with the concept. The CPA approach is not new. Bruner (1966) is credited for first suggesting that pupils learn through different modes or representations:

1. Enactive representation (action-based); now more commonly known as concrete;
2. Iconic representation (image-based); now more commonly known as pictorial;
3. Symbolic representation (language-based); now more commonly known as abstract.

Following on from Bruner there is a wealth of research and evidence that support this teaching and learning approach in mathematics:
*"Research has shown that the optimal presentation sequence to teach new mathematical content is through the (CPA) approach. (*Sousa, 2008)

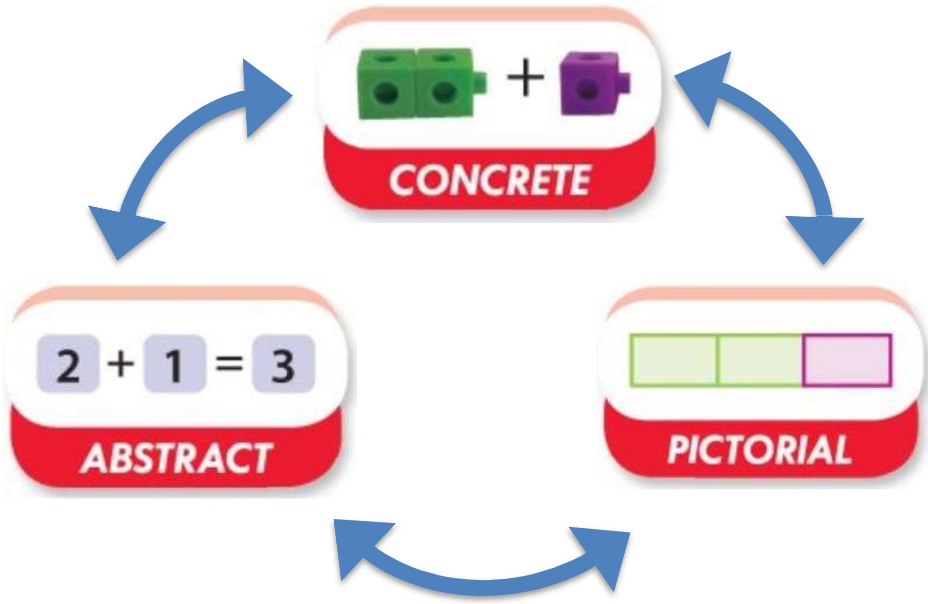
*"Students who use concrete materials develop more precise and more comprehensive mental representations, often show more motivation and on-task behaviour, understand mathematical ideas, and better apply these ideas to life situations" (*Hauser, 2009)

*"I utilize the CPA approach to foster a deeper understanding of mathematics so that students are gaining greater conceptual knowledge rather than procedural knowledge. Through this approach, students are experiencing and discovering mathematics rather than simply regurgitating." (*Gujarati,2013)

Children don't learn mathematical knowledge through being told things that they just have to memorise. They construct mathematical knowledge through exploring, discovering, researching and using. Mathematical representations (or manipulatives) demonstrate the move from physical representations of mathematics to using mental images and finally to more abstract mathematics.

The aim for children is that they use manipulatives and resources during their mathematical learning experiences.

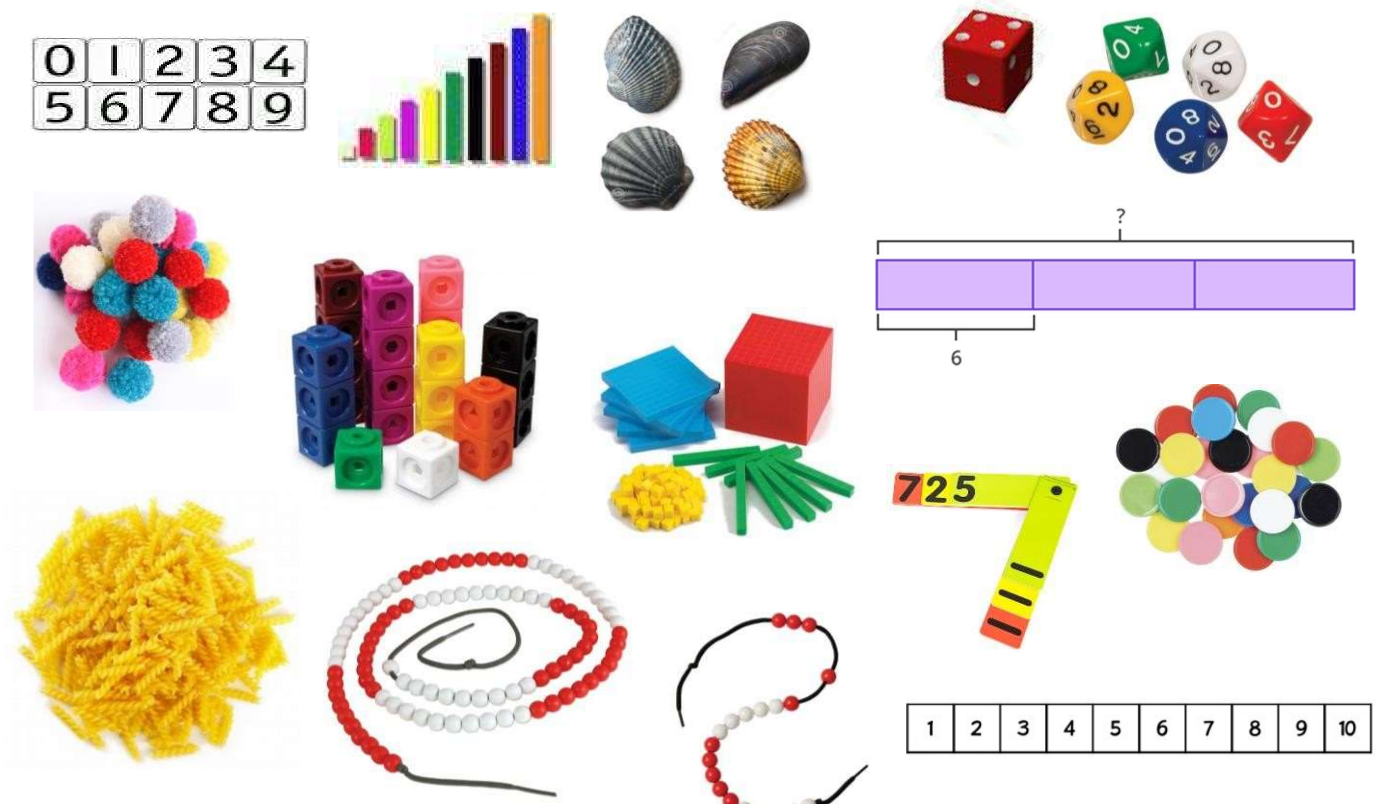
Although in early teaching of mathematics, pupils might be exposed to experiences from a concrete to pictorial then abstract approach, this model can be used flexibly and in any order.

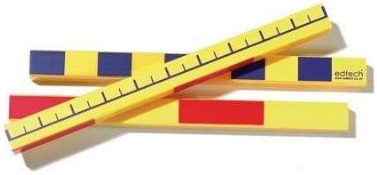


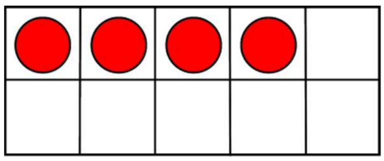
Children have the opportunity to manipulate a wide variety of models, images and resources to choose the best representation for each calculation.

Representations are a significant aid in developing conceptual understanding. Different concepts should be represented using the same resource/representation.

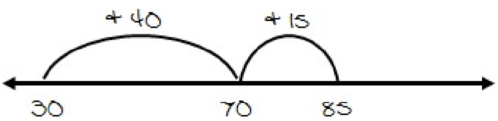
These could include:
Numicon, number lines, number fans, bead strings, counters, counting objects, cubes, Dienes, Cuisenaire rods, multilink, unifix, place value cards, 100 square, dice, arrow cards, digit cards, counting sticks, bar models, Gattengno chart, ten frames, money, scales, etc.







0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1000	2000	3000	4000	5000	6000	7000	8000	9000



Language

Children who learn to explain why something makes sense and reason through their mathematical explanations will develop a deeper knowledge that will result in long-term understanding. We want our children to be able to explain their strategies confidently using the correct vocabulary.

It is essential that all children are exposed to and supported in developing quality and varied mathematical vocabulary. This understanding will consequently support children in their ability to access problems, present mathematical conjecture and justification through reasoning.

A definition of mathematical terms may be found in the Glossary.

Equals = equal to, the same as, equivalent

Addition	Subtraction
add, addition, more, plus, increase, and, make, sum,	subtract, subtraction, less/fewer, take (away), minus,
total, altogether,	decrease, difference between, leave
double,	half, halve,
one more, two more, ten more etc…	one less, two less… ten less etc…
part and whole	part and whole
addend, augment	minuend, subtrahend
equivalent, equals to, is the same as, inverse	equivalent, equals to, is the same as, inverse
Multiplication	**Division**
multiply, multiplication, lots of, groups of	divide, division, divided, lots of, groups of, share, sharing
multiplied by	divided by, into, divisible by, remainder, left,
multiple of, product	factor, quotient,
double, doubling	halve, halving
once, twice, three times etc…	one each, two each, three each etc…
repeated addition, multiplicative	repeated subtraction
array, row,	array, row,
scaling	scaling
equivalent, equals to, is the same as, inverse	equivalent, equals to, is the same as, inverse

The Calculations Working Party

The following schools and teachers were involved in the planning and writing of this guidance in February 2020.

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